2.3 Geometrical optics

Basic theory

Fresnel theory of diffraction is simple, but using it, we can analyze thin planar obstacles only. General theory, which was described in chapter 2.2, is formally rather complicated, and only geometrically simple objects can be handled with. Therefore, alternative ways of the analysis were sought out. *Geometric theory of diffraction* (GTD) belongs to those ways: GTD numerically computes even rather complicated situations. Before explaining the matter of GTD, the basic terms of *geometrical optics* (GO) are introduced to the reader.

Today's geometrical optics is an efficient tool for solving wave phenomena (wave propagation) in complex media. GO is not limited to the range of optical frequencies, and it can be used even for radio waves. From the classical geometrical optics, the idea of wave propagation along beams was adopted. Moreover, GO is able to compute not only wave trajectories but too changes of field intensities and polarization of waves during propagation. The theory of GO is based on the following two assumptions:

1. Wavelength is small, and therefore, wave number k is high.
2. The wave is observed far away from the source. Whereas the wave amplitude changes slowly in the propagation direction, phase varies quickly. The sense of this requirement can be perceived using the following illustration example.
   We are interested in the propagation of the spherical wave in the distance of 10 wavelengths from the source. If the distance is increased for one half of the wavelength, i.e. for 5 %, the intensity amplitude decreases for 5 % too, but the phase changes for π radians (a significant change).

We start the explanation of geometrical optics by modifying the relation for the intensity of electromagnetic field. Instead of $E = E_m \exp(-jkr)$, we write

$$E = E_m \exp\left[-jk_0 L(x,\, y,\, z)\right]. \qquad (2.3A.1)$$

In the exponent, we have in all the situations $k_0 = \omega\ (\varepsilon_0\ \mu_0)^{1/2}$ and the parameters of the medium are included in the function L. We simply understand that $L(x,\, y,\, z) = const$ is equation of equiphase surface (wave surface) and that the vector grad L is of the direction, which is perpendicular to equiphase surface, i.e. of the propagation direction.

The relation (2.3A.1) is substituted to Maxwell equations. Assuming that the wave number k is high, relatively complicated rearrangements yield

$$|grad\, L|^2 = n^2, \qquad (2.3A.2)$$

where

$$n = k / k_0 = \sqrt{\varepsilon_{rel}\mu_{rel}} \qquad (2.3A.3)$$

denotes the refractive index of the medium.

Eqn. (2.3A.2) is called *the basic equation of geometrical optics*. The function $L(x,y,z)$ is called the eiconale. It is the scalar function of coordinates. The vector grad L is of the direction of spherical wave propagation in every point. The curve, which tangent is of the direction of grad $L$is every point, is called the beam. The beam is of the direction of the steepest change of phase in every point, and it is of the direction of Poynting vector too (i.e. of the direction of the energy flow). In an inhomogeneous medium, beams can be curved and eqn. (2.3A.2) is the differential equation of beams.

For practical computations of beams, the form of (2.3A.2) is not suitable. Therefore, the following relations are used for computing beam trajectories:

$$\frac{\partial}{\partial s}\left(n\frac{\partial x}{\partial s}\right) = \frac{\partial n}{\partial x},\quad \frac{\partial}{\partial s}\left(n\frac{\partial y}{\partial s}\right) = \frac{\partial n}{\partial y},\quad \frac{\partial}{\partial s}\left(n\frac{\partial z}{\partial s}\right) = \frac{\partial n}{\partial z}, \qquad (2.3A.4)$$

$$\left(\frac{\partial x}{\partial s}\right)^2 + \left(\frac{\partial y}{\partial s}\right)^2 + \left(\frac{\partial z}{\partial s}\right)^2 = 1. \qquad (2.3A.5)$$

The variable s is curvilinear coordinate *along* the beam. Details are given in the layer B including the derivation and an illustrative example.

Geometrical optics enables to compute not only beam trajectories but too the variations of amplitude and phase of field intensity along the beam:

In the starting point (A e.g.) a (infinitely) facet dS_1 is chosen of the wave surface and a beam is led through every point of the edge of this facet. That way, a *beam tube* is obtained. On some of the following equiphase surfaces (B), the beam tube is of the different cross section dS_2 (fig. **2.3A.1**). Since the energy propagates *along* the beams, it cannot leave the tube through the side walls. In the lossless medium, the power passing facets dS_1 and dS_2 is identical. Since $P = \Pi\ S = (E^2/Z_0)\ S$ and $Z_0 = (\mu/\varepsilon)^{1/2}$, we can simply derive the relation between intensities on both the facets:

$$\sqrt{\frac{\varepsilon_1}{\mu_1}}|E_1|^2 dS_1 = \sqrt{\frac{\varepsilon_2}{\mu_2}}|E_2|^2 dS_2. \qquad (2.3A.6)$$

Phase of field intensity in B can be computed using eiconale, resp. using eqns. (2.3A.1) or (2.3A.2). If the eiconale is of the value L_A at the beginning of the trajectory A, then in B (which has to be located at the same beam)

$$L_B = L_A + \int_A^B n\, ds. \qquad (2.3A.7)$$

Integration is done along the beam.

Eqn. (2.3A.6) is not valid in regions, where the beams cut (infinitely high field intensity would be obtained). Such situation can be met in the focus and on the surface called *caustics* (see layer B).

In more complicated cases, beams in different transversal planes are of different curvature radii of their wave surfaces. In such situations, (2.3A.6) is not valid. Nevertheless, the intensity can be computed (see layer B).

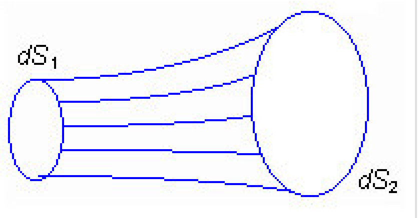

Fig. 2.3A.1 *Beam tube*