6.1 Analysis of frequency selective surfaces

Advanced theory

In this part of the textbook, we take a detailed look on basis functions, which can be used when numerically analyzing frequency selective surfaces (current distribution over an element is sought).

In the layer A, we derived a relation for electric field of a frequency selective surface, which consists of metallic elements, in the domain of spatial spectra

$$-\frac{1}{2\omega\varepsilon}\sum_{m,n}\begin{bmatrix}\frac{k^2-\alpha_m{}^2}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}} & \frac{-\alpha_m\beta_n}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}}\\ \frac{-\alpha_m\beta_n}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}} & \frac{k^2-\beta_n{}^2}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}}\end{bmatrix}\begin{bmatrix}J_x(\alpha_m,\beta_n)\\ J_y(\alpha_m,\beta_n)\end{bmatrix}\exp[j(\alpha_m x+\beta_n y)] = -\begin{bmatrix}E_x^I(x,y)\\ E_y^I(x,y)\end{bmatrix}. \qquad (6.1B.1)$$

In the above-given relation, ω denotes angular frequency of a wave, ε is permittivity of the surrounding of selective surface, α_m and β_n are spatial frequencies, k denotes free-space wave number, J_x and J_y are components of current density vector over the element, and finally, $E_x{}^I$ and $E_y{}^I$ denote components of electric field intensity of incident wave.

Solving the problem, we approximate the unknown current distribution in (6.1B.1) in terms of properly chosen basis functions and unknown approximation coefficients. Such formal approximation is substituted to the solved equation (6.1B.1). Since the approximation does not meet the solved equation exactly, we respect this fact by introducing a residual function (residuum). Smaller values the residual function reaches, more accurate the approximation is. The residuum is minimized by Galerkin method (residuum is sequentially multiplied by as many basis functions as many unknown approximation coefficients have to be computed; that way the set of N linear algebraic equations for N unknown approximation coefficients is obtained).

There are two approaches for choosing basis functions. The first one exploits basis functions, which are non-zero over the whole analyzed area. Functions are elected to physically represent standing waves of a current on an element.

The second approach divides the analyzed region to sub-regions, where current is approximated in terms of basis functions, which functional value is non-zero over a given sub-region only. This approach is advantageous in simple analysis of selective surfaces consisting of arbitrarily shaped elements.

Harmonic basis functions

Assume a frequency selective surface consisting of rectangular perfectly conductive elements of the dimensions a' and b'. Components of current density J are approximated as follows **[17]**

$$\mathbf{J}(x,y) = \sum_{p=0}^{P}\sum_{q=0}^{Q} A_{pq}\Psi_{pq}^{TM} + F_{pq}\Psi_{pq}^{TE}, \qquad (6.1B.2)$$

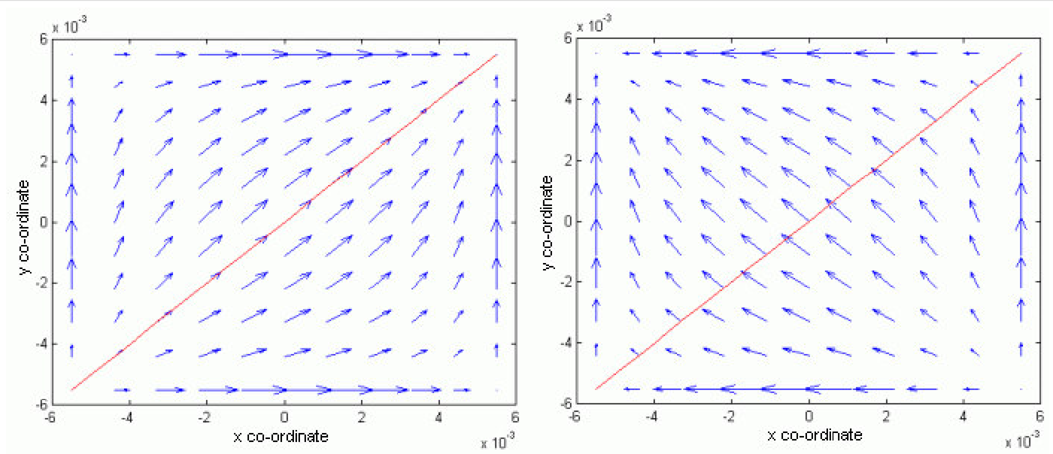
where

$$\begin{aligned}\Psi_{pq}^{TE}(x,y) &= \left[\frac{p\pi}{a'}\sin\left(\frac{p\pi x}{a'}\right)\cos\left(\frac{q\pi y}{b'}\right)\mathbf{u}_x + \frac{q\pi}{b'}\cos\left(\frac{p\pi x}{a'}\right)\sin\left(\frac{q\pi y}{b'}\right)\mathbf{u}_y\right]\exp[j(\alpha_0 x+\beta_0 y)],\\ \Psi_{pq}^{TM}(x,y) &= \left[\frac{p\pi}{b'}\sin\left(\frac{p\pi x}{a'}\right)\cos\left(\frac{q\pi y}{b'}\right)\mathbf{u}_x - \frac{q\pi}{a'}\cos\left(\frac{p\pi x}{a'}\right)\sin\left(\frac{q\pi y}{b'}\right)\mathbf{u}_y\right]\exp[j(\alpha_0 x+\beta_0 y)].\end{aligned} \qquad (6.1B.3)$$

Here, $\Psi_{pq}{}^{TE}$ and $\Psi_{pq}{}^{TM}$ are two different functions for parallel polarization and perpendicular one. These functions differ in two aspects:

- Indeces p, q for the function $\Psi_{pq}{}^{TE}$ can reach zero, and for $\Psi_{pq}{}^{TM}$ are one and higher.
- Component J_y for perpendicular polarization has to be of the opposite sign than J_y for parallel polarization.

In order to get better notion about the difference between currents, which are excited by electric intensity of incident wave, which is parallel (perpendicular) to the plane of incidence, we depict directional fields of current density of mode (1,1) for parallel polarization and perpendicular one in fig. **6.1B.1**.



a) b)

Fig. 6.1B.1 *Directional field of current density of the mode (1,1)*
***a)** for parallel polarization*
***b)** for perpendicular one.*
The cell is of dimensions a = b = 12mm, the metallic element is of dimensions a′ = b′ = 12mm; f=10GHz, ϑ = 1°, φ = 45°, d = 1.5mm, ε_r = 3.7. Magnitude of marginal currents was increased with respect to reality. Plane of incidence is depicted in red.

Combination of harmonic functions and Chebyshev polynomials

$$\mathbf{J}^{TE} = \sin\left[\frac{p\pi}{a'}\left(x+\frac{a'}{2}\right)\right]\frac{T_q(2y/b')}{\sqrt{1-(2y/b')^2}}\mathbf{x} + \frac{T_p(2x/a')}{\sqrt{1-(2x/a')^2}}\sin\left[\frac{q\pi}{b'}\left(y+\frac{b'}{2}\right)\right]\mathbf{y} \qquad (6.1B.4a)$$

$$\mathbf{J}^{TM} = \sin\left[\frac{p\pi}{a'}\left(x+\frac{a'}{2}\right)\right]\frac{T_q(2y/b')}{\sqrt{1-(2y/b')^2}}\mathbf{x} - \frac{T_p(2x/a')}{\sqrt{1-(2x/a')^2}}\sin\left[\frac{q\pi}{b'}\left(y+\frac{b'}{2}\right)\right]\mathbf{y} \quad (6.1B.4b)$$

In contrast to harmonic basis functions, the function cosine is replaced by *Chebyshev polynomial* in order to reach infinite values at the edges of elements. That way, currents flowing along edges are correctly modeled.

Now, eqn. (6.1B.1) is solved by Galerkin method: eqn. (6.1B.1) is sequentially multiplied by basis functions and the product is integrated over the surface of the element. For purely harmonic basis functions, we get:

$$\int_a^b w_m(z)R(z)dz = 0 \quad m = 0,1,\ldots N,$$

$$\sum_{p,q} A_{p,q}\left\{\sum_{m,n}\Psi_{rs}^{TE*}(\alpha_m,\beta_n)K(\alpha_m,\beta_n)\Psi_{pq}^{TE*}(\alpha_m,\beta_n)\right\} + F_{pq}\left\{\sum_{m,n}\Psi_{rs}^{TE*}(\alpha_m,\beta_n)K(\alpha_m,\beta_n)\Psi_{pq}^{TE*}(\alpha_m,\beta_n)\right\} = \frac{-1}{ab}\iint_{p\check{r}es\,element} E_{\tan}^{I}(x,y)\Psi_{pq}^{TE*}(x,y)dxdy, \quad (6.1B.5a)$$

$$\sum_{p,q} A_{p,q}\left\{\sum_{m,n}\Psi_{rs}^{TM*}(\alpha_m,\beta_n)K(\alpha_m,\beta_n)\Psi_{pq}^{TM*}(\alpha_m,\beta_n)\right\} + F_{pq}\left\{\sum_{m,n}\Psi_{rs}^{TM*}(\alpha_m,\beta_n)K(\alpha_m,\beta_n)\Psi_{pq}^{TM*}(\alpha_m,\beta_n)\right\} = \frac{-1}{ab}\iint_{p\check{r}es\,element} E_{\tan}^{I}(x,y)\Psi_{pq}^{TM*}(x,y)dxdy. \quad (6.1B.5b)$$

If the above-given set is solved out, electric intensity $\mathbf{E}^S$ in spectral domain is computed. Intensity $\mathbf{E}^S$ appears at the left-hand side of eqn. (6.1B.1)

$$\mathbf{E}^S = -\frac{1}{2\omega\varepsilon}\sum_{m,n}\begin{bmatrix}\frac{k^2-\alpha_m{}^2}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}} & \frac{-\alpha_m\beta_n}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}}\\ \frac{-\alpha_m\beta_n}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}} & \frac{k^2-\beta_n{}^2}{\sqrt{k^2-\alpha_m{}^2-\beta_n{}^2}}\end{bmatrix}\begin{bmatrix}J_x(\alpha_m,\beta_n)\\ J_y(\alpha_m,\beta_n)\end{bmatrix}\exp[j(\alpha_m x+\beta_n y)] = -\begin{bmatrix}E_x^I(x,y)\\ E_y^I(x,y)\end{bmatrix}. \quad (6.1B.6)$$

Computing electric intensity of scattered wave $\mathbf{E}^S = \mathbf{E}^R$, only the basic harmonic component on the frequency α_0, β_0 is assumed. If the period a is small (smaller than one half of the wavelength) and frequency approaches the first resonance of the surface, then a uniform field of the scattered wave exists in the far field region only. If the condition is not met, then parasitic modes can be excited (higher spatial frequencies are not filtered by the near-field zone, and corresponding waves propagate further).

Let us explain the physical notion of parasitic modes. Explanation is illustrated by fig. **6.1B.2**. Here, a two-element antenna array is depicted, which is illuminated by a plane wave arriving from the direction ϑ. After the incidence, the wave is reflected according to rules of geometric optics. A part of energy can propagate even in the direction of the parasitic (grating) lobe.

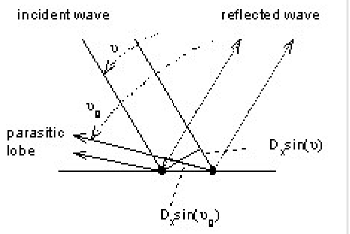


Fig. 6.1B.2 *Explanation of forming parasitic lobes (two-element antenna)*

Formation of a parasitic lobe is conditioned by reaching a phase shift, which equals to the integral multiple of 2π by the beam no. 1 on a bold trajectory. From the mathematical point of view:

$$\beta D_x \sin(\vartheta_g) + \beta D_x \sin(\vartheta) = n2\pi. \quad (6.1B.7)$$

The direction, where the parasitic lobe appears, depends on the frequency of the incident wave and on the distance of antenna elements only. The lowest frequency of exciting the parasitic mode for a given angle of incidence, is for $\vartheta_g = 90°$:

$$f_{g0} = \frac{nc}{D_x(\sin\vartheta + 1)}. \quad (6.1B.8)$$

if the distance of antenna elements equals to $a = 21$ mm and if perpendicular incidence of wave are assumed, then the first parasitic lobe is excited on the frequency

$$f_g = \frac{1\cdot 3\cdot 10^8}{0{,}021\cdot(\sin 0+1)} = 14{,}3\,GHz. \quad (6.1B.9)$$