3. Spread Spectrum Systems

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Spread spectrum signals are signals carrying digital modulation as a rule. What signal has a spread spectrum? The specific parameter is the ratio of a signal bandwidth to the transmission rate. The bandwidth of a spread spectrum signal is a multiple of the bandwidth of a modulating data signal (i.e. of its data rate). PSK or QAM modulated signal has the spectrum whose bandwidth is roughly equal to the value of the modulation (symbol) rate of the applied digital modulation. In this case the ratio of its spectrum bandwidth to the data transmission (modulation) rate reaches a value in the vicinity of one. In contrast, a signal has its spectrum spread if this ratio approaches a value substantially (often by some orders of magnitude) higher than one. As a rule, to reach such a broad spectrum, an additional modulation by a spreading signal is applied. Spreading signal does not carry any information but only one determining the transmitting user. The spreading modulation is a periodic deterministic digital signal, referred frequently to as a signature, generated by means of a code sequence (binary or nonbinary).

The receiver of a spread spectrum signal first eliminates the spreading modulation. As the elimination of the spreading modulation of a received signal is the first step of the signal processing performed by a receiver, the legitimate question can be raised. What is the reason of the spectrum spreading if we aim at its elimination again?

Motivation
Arguments for application of the spread spectrum techniques are connected to the character of an explored communication channel. We would lack arguments if the communication over an AWGN channel took place. This is the case of a single-path time-invariant interference-free communication channel. Any transmitted signal is corrupted by an additive white Gaussian noise only and the general expression for an error rate of the transmission behaves according to the well-known rule expressing the error rate as a function of the SNR and a correlation of symbols only. The shape of an applied signal in the time domain and, equivalently, the shape of its spectrum do not influence the value of the error rate. There is no reason to employ a spread spectrum technique for transmission of signals in such environment.

The opposite conclusion is reached if the utilized channel is a highly signal disturbing one. This class contains channels such as
- channels with interference
- channels exhibiting time-variant and frequency-selective parameters
- channels with multi-path propagation.

Spread spectrum signals are advantageous not only in communications. As broadband signals, spread spectrum signals potentially enable precise localization in the time domain, which facilitates precise measurement of time intervals, and hence even distances and positions of objects.

Spread spectrum signals exhibit a low value of their mean value of the power spectrum density (PSD) and potentially even its peak value. The detection of such signals by methods evaluating the ratio of the signal power to the additive noise power in the band-pass of a
receiver is made difficult in a hidden area, where the noise PSD has higher value compared to the transmitted spread spectrum signal. The complementary area in which a SS signal is not hidden is a circle around the position of the transmitter.

**Methods of spectrum spreading**

Any wide-band spread spectrum signal belongs to one of the three categories:

- signals without a carrier
- signals with a single carrier
- signals with multiple carriers.

**Signals without a carrier** are relatively new subclass of wide-band signals. These signals are created by a sequence of very narrow pulses of a proper shape and their spectrum bandwidth is by some orders of magnitude wider than the modulation rate. Such signals are often referred to as Ultra Wide-Band (UWB) signals (and related UWB systems). Their bandwidth can reach values higher than 5 GHz [4].

The UWB principle is not limited to radio-communication systems only, but it proved advantageous in optical-communication systems, as well. As a rule, the modulation applied here is a pulse-position modulation or a spreading coded modulation.

**Single harmonic carrier signals** create the basic category of SS signals generated by modulating a harmonic carrier. This category can be divided into three basic subgroups.

- **Direct sequence (DS) SS signal** is generated by PSK spreading code modulation. If the BPSK is used, this modulation is equivalent to the keying of the polarity of the transmitted carrier. QPSK can equivalently be regarded as a polarity keying of both quadrature components. The BPSK modulated signal of the k-th user can be expressed as

\[ s_k(t) = A_k \cdot c_k(t) \cdot b_k(t) \cdot \cos(\omega_0 t + \phi_{ak}) , \]  

where \( A_k \) is the amplitude of the signal, \( c_k(t) \) is a spreading code modulation, \( c_k(t) = \pm 1 \). Let the shaping function \( p_o(t) \) be defined as a rectangular one in our case. Using the unit step function \( 1(t) \), this function is \( p_o(t) = 1(t) - 1(t-\theta) \). The spreading modulation \( c_k(t) \) can be expressed as

\[ c_k(t) = \sum_i c_{ki} p_{ci}(t-iT_c), \]  

where \( T_c \) is the bit interval of this modulation referred to as the chip interval. The sequence \( c_{ki} \), \( i = \ldots, -1, 1, \ldots \) is a periodic one having period \( L \), i.e., a spreading modulation (signature) \( c_k(t) \) is periodic as well

\[ c_k(t) = c_k(t - i \cdot L \cdot T_c), \]  

\[ \text{i integer} \]  

The data modulation \( b_k(t) \) carries a binary data sequence \( \{b_{ki}\}, i = 0, 1, 2, \ldots \)

\[ b_k(t) = \sum_i b_{ki} p_{ki}(t-iT), \]  

where \( T \) is the data (symbol) interval. The particular case when this interval equals to the spreading code modulation period

\[ T = L \cdot T_c \]  

is called “short code spreading”. The case of the spreading modulation period much longer than the data interval is referred to as the long code spreading. In the former
case the bandwidth of SS signal is increased L-times compared to the data only modulated signal as the signal bandwidth of the signal before spreading is $\sim \frac{1}{T}$ and that after spreading is $\sim \frac{1}{T_c} = \frac{L}{T}$.

The value $L$ is often referred to as the system (or spreading) gain.

- **Frequency hopping** (FH) is spectrum spreading by step-wise changes of the carrier frequency. The carrier is data modulated by any digital modulation as FSK, PSK or QAM. The slow FH is the case when the data symbol interval is a fraction of the (chip) interval between two adjacent frequency steps. The case of multiple frequency steps within one data symbol interval is referred to as the fast FH. The sequence of frequency values which are hopped over is usually created as an equidistant set of frequencies, the frequency step being higher than the bandwidth of the spectrum of the data modulated signal before spreading. It avoids the case when two adjacent frequency positions are interfered by a single narrow-band (line) interference because their spectra overlap. This method is effective in the case of a small number of narrow-band interferences. The system can be designed as adaptive, which means that it is able to exclude instantly interfered frequencies from the set of frequencies the system is hopped on. The sequence of these frequencies is controlled by a code sequence with proper characteristics.

- **Time hopping** (TH). The controlling code sequence defines a position of each short interval within which the accumulated data are transmitted as a packet. The rate of this modulation within a packet is a multiple of the mean value of data transmission. As a rule this multiplicity is one or more orders of magnitude. The “randomness“ of the packet position avoids the dangerous regularity of interference in the “worst case“.

Among **multi-carrier signal formats**, it were the promising parameters of orthogonal frequency division multiplexing (OFDM) that attracted the communication system designers. This case is characterized by transmitting a high number of carriers with equidistant frequencies. This distance is chosen with respect to the orthogonality of carriers on a data or chip interval, respectively. The input binary data sequence is serial-to-parallel converted and transmitted through low-rate, frequency overlapping but orthogonal sub-channels. The narrow-band nature of signals generated by modulation of carriers provides high immunity against multipath dispersion and narrow band interference. Any application of OFDM consisting of a higher number of carriers would not be possible without application of DSP to signal generation and detection. Majority of application uses Fast Fourier Transform (FFT) transformers realized in a form of integrated circuits (IC). OFDM in itself does not cause signal spectrum spreading. This spreading is reached by additional frequency hopping or more frequently by direct sequence spreading. The latter method is usually referred to as multicarrier CDMA.

The application of CDMA spreading can be done by multiplication of i-th modulated carrier of k-th user (frequency $f_i = f_1 + (i - 1) \Delta$ ), by a value $c_{ik}$ of a chosen spreading code. The spreading code length equals the number of carriers. The data stream is not subjected to any serial-to-parallel conversion; the same data bit modulates all the carriers. This method is often referred to as Frequency domain spreading. The alternative method uses data sequence serial-to-parallel conversion and all modulated carriers are spread by the same spreading modulation. The frequency distance $\Delta$ is chosen such that the carriers are orthogonal on the chip interval $T_c$, i.e., the orthogonality condition $T_c \cdot \Delta = n$, n positive integer, holds.
Transient methods between these two different methods combining features of both of them are reported in [8].

The last subclass of MC-CDMA utilizes S/P conversion, the identical spreading modulation in all channels and the frequency distance $\Delta$ is chosen to satisfy orthogonality on the data symbol interval.

Orthogonality in time and frequency domains

Time-domain orthogonality

Two signals $x_1(t)$ and $x_2(t)$ are orthogonal on the interval $t \in (a,b)$, $a<b$, $a,b$ real, if and only if

$$\int_a^b x_1(t) x_2(t) \, dt = 0 \quad (6)$$

In the case of digital communications, $x_1(t)$ and $x_2(t)$ are usually real and time-limited signals having interval of nonzero values $0,T$, i.e., $a=0$, $b=T$.

Frequency domain orthogonality

The question about the relation of orthogonality in the time and frequency domains is answered by the Parseval theorem, the common shape of which is expressed as

$$\int_{-\infty}^{\infty} x_1(t) x_2(t) \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\omega) X_2(\omega) \, d\omega \quad (7)$$

Here, $X_1(\omega)$ and $X_2(\omega)$ are Fourier transforms of (finite energy) signals $x_1(t)$ and $x_2(t)$, respectively. If both signals have fully or mostly disjoint (nonoverlapping) spectra, the signals are orthogonal or weekly correlated, respectively. This can be seen as a mechanism of orthogonality or weak correlation between a broadband (i.e., including SS) and a narrowband signal. Any weekly correlated pair of signals can be discriminated by a correlator. That mechanism is utilized in any kind of multiplexing of signals including code division multiplex / code division multiple access - CDMA.

The number of mutually orthogonal signals is limited by the dimensionality of the utilized signal space. The double of product of the utilized frequency bandwidth $B$ and the length $T$ of orthogonality interval, i.e. the symbol interval in our case, give dimensionality. This dimensionality does not suppose any limitation on a class of functions creating an orthogonal basis. If we confine ourself on e.g. orthogonal binary signals, their number will usually not reach the signal space dimensionality.

Advantages and potential of SS communication

Interference suppression

Signal spreading was originally recognized as a powerful tool for suppression of narrow band interferences. Generally, correlator/matched filter receiver can suppress any weakly correlated interference. The lower the correlation, the higher the interference suppression. The degradation by a narrow-band interference can be eliminated by means of the synergy of an OFDM multicarrier signal and a forward error control coding.

A signal hiding in additive noise is a consequence of an increase of the spectrum width given by a spreading factor equal to one, two or more orders of magnitude, respectively, keeping the total power of this signal unchanged. Therefore, the values of the power spectral density are lowered inversely proportionally to the spreading factor. To keep the power
spectral density low in the whole bandwidth of spreading, the applied spreading modulation has to have corresponding property.

**Multipath propagation**, which can cause severe signal distortion, is mitigated by structures based on the Rake receiver [1], [2], [3]. If mutually delayed signal components can by resolved by correlator/matched filter, the receiver processes the delayed signal replicas separately. This resolution can be done, if the differences of signal components delays are higher than the width of the signal autocorrelation function main lobe. This has to be taken into account when designing the spreading code modulation.

**Intersymbol interference** (ISI) caused by multipath propagation can be reduced by increasing the length of the data symbol interval. One of possible ways to do it is the application of the OFDM principle, where the data symbol interval is lengthened proportionally to the number of carriers. Mitigation of **selective fading** in frequency selective channels can be done by two different methods. The first is based on the idea that this fading will not cause a severe deterioration in OFDM if the number of faded sub channels is limited and the signal drop-outs can be eliminated by the FEC (Forward Error Correction) coding. Interleaving can increase the robustness of this method. The second method relies on **frequency diversity**, which is a mechanism leading to lower degradation of a signal whose bandwidth is substantially broader than the bandwidth of the fading notch. This is an area of implementation of spread spectrum onto OFDM. Applying the OFDM-CDMA methods can mitigate combination of selective fading and multipath propagation.

**Resistance to selective fading**

Behavior of a channel in the frequency domain can be described in terms of the channel coherence bandwidth defined as the maximum value of the bandwidth in which the values of the channel transfer function (as a function of frequency) are strongly correlated. A channel whose bandwidth of coherence is lower than the spectrum width of a transmitted signal is referred to as a frequency selective channel. If the opposite is true, the channel is said to be frequency nonselective. The same channel can act as frequency selective, when a wide band signal is transmitted, as well as a frequency nonselective, when a narrow band signal is transmitted. The bandwidth of a transmitted signal determines signal distortion. In general, a total outage of the signal at the output of a frequency nonselective channel has more severe consequences than the signal distortion caused by a frequency selective channel. To change the signal canceling frequency nonselective fades to the less catastrophic frequency selective ones, one has to broaden the bandwidth of a transmitted signal. This calls for application of a spread spectrum technique.

To visualize a signal distortion by a frequency selective channel, a simple - though unpublished - “worst case” approximation is given below. The DS-SS signal $s(t)$ (1) passes through a frequency selective channel whose transfer function $H(j\omega)$ exhibits one deep rectangular fade of bandwidth $\Delta\omega = 2\pi B$ located around $\omega_0$. The rectangular shape is less favourable than any real frequency selective fade). This band-stop transfer function $H(j\omega)$ can be expressed as

\[
H(j\omega) = \begin{cases} 
1 & \text{for } |\omega| \notin (\omega_0 - \pi B, \omega_0 + \pi B) \\
0 & \text{for } |\omega| \in (\omega_0 - \pi B, \omega_0 + \pi B)
\end{cases}
\]

or

\[
H(j\omega) = 1 - H_1(j\omega)
\]

where
\[ H_1(j\omega) = \begin{cases} 
1 & \text{for } |\omega| \in \left(\omega_0 - \pi B, \omega_0 + \pi B\right) \\
0 & \text{for } |\omega| \notin \left(\omega_0 - \pi B, \omega_0 + \pi B\right) 
\end{cases} \quad (10) \]

The impulse response of this channel is an inverse Fourier transform, which is a superposition of the impulse response \( \delta(t) \) of ideal linear non band-limited channel and of the impulse response of a band-stop rectangle \( h(t) \)

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega)e^{j\omega t}d\omega = \delta(t) - h(t) \]

where

\[ h_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_1(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi} \int_{-\omega_0 - \pi B}^{-\omega_0 + \pi B} e^{j\omega t}d\omega + \frac{1}{2\pi} \int_{\omega_0 - \pi B}^{\omega_0 + \pi B} e^{j\omega t}d\omega = 2B\cos\omega_0 t \cdot \frac{\sin \pi B t}{\pi B t} \]

The relevant component of the channel response is

\[ y_1(t) = s(t) * h_1(t) = 2B \int_{-\infty}^{\infty} \cos \omega_0 \tau \cdot \frac{\sin \pi B \tau}{\pi B \tau} c(t-\tau) \sin \left[ \omega_0 (t-\tau) + \varphi_0 \right] d\tau \quad (11) \]

The substitution (2) leads to

\[ y_1(t) = B \sin(\omega_0 t + \varphi_0) \sum_i c_i \int_{-\infty}^{\pi B(t-iT_c)} \frac{\sin \pi B \tau}{\pi B \tau} p_T (t-\tau-iT_c) d\tau = \]

\[ = \frac{1}{\pi} \sin(\omega_0 t + \varphi_0) \sum_i c_i \int_{\pi B(t-iT_c)}^{\pi B(t-(i+1)T_c)} \frac{\sin \vartheta}{\vartheta} d\vartheta \quad (12) \]

This response is a superposition of responses to individual chips. The response to the positive chip is

\[ y_1(t) = \frac{1}{\pi} \sin(\omega_0 t + \varphi_0) c_i \int_{\pi B(t-(i+1)T_c)}^{\pi B(t-iT_c)} \frac{\sin \vartheta}{\vartheta} d\vartheta \quad (13) \]

A graph of the function \( \frac{1}{\pi} \int_{\pi B(t-(i+1)T_c)}^{\pi B(t-iT_c)} \frac{\sin \vartheta}{\vartheta} d\vartheta \) is given in Fig.1.

As this response has the interval of nonzero values some tens of symbol intervals long, the notch component will cause intersymbol interference (ISI), not necessarily severe. The complete fading of a signal in a nonselective faded channel is converted to a relatively long but weak ISI component induced by this fade. The narrower the bandwidth of a fade the longer and weaker this component is. The positive influence of the frequency diversity is obvious.

![Graph of the function](image)

Fig.1. Response of the notch to single rectangular chip
**Spreading coded modulation – signatures**

**Requirements**
- The spread spectrum signal should have the envelope constant to keep the sensitivity to nonlinearities low
- Applications of binary and quaternary PSK modulations are prevailing, i.e., attention is concentrated on binary and quaternary sequences
- Smooth spreading – as uniform power density in the spreading bandwidth as possible without discrete lines in the spectrum
- Narrow main lobe and very low side lobes of the autocorrelation function (for simple and unique code synchronization and delay measurements)
- Low cross-correlations within the set of CDMA signatures

The character of a specific application determines the proper definition of the correlation functions. The well-known [1], [2], [3] optimized sequences determining spreading modulation-signature are pseudorandom m-sequences are e.g. Gold sequences, Kasami sequences, bent sequences, Walsh-Hadamard sequences and many others; in the meantime even some non binary ones have been proposed. Their correlation properties have been thoroughly studied.

**Receiver structure**

Taking into account complexity of a spread spectrum signal receiver, the class of receivers is usually limited to the linear one. The optimum receiver of a signal having a priori known shape in AWGN channel is a correlator or its equivalent - matched filter (MF) [1], [2], [3]. This receiver is an optimum solution even in the case of discrimination of orthogonal signals in AWGN channel [5]. This solution enables a resolution of a number of mutually orthogonal signals in such a way, that the response of MF/correlator is a function of amplitude (energy) of a signal the MF is matched to (target signal), and it is completely independent of amplitudes of signals at the MF input which are orthogonal to this target signal. The signal discrimination is ideal in this case. If the detected target signal is corrupted by the superposition of non-orthogonal signals, the MF response has an interfering component proportional to the correlations with the target signal and amplitudes of the interferers. The optimality of MF is not valid in this environment any more; the optimum linear receiver is a receiver maximizing the value of signal to interference plus noise ratio. The sensitivity of detector response to amplitudes of interferences is referred to as near-far problem [5].

**Code Division Multiplex / Code division Multiple Access (CDMA)**

When a channel is shared by a number of users, it can be taken as a special case of interfering signals in this channel. The methods of multiplexing of signals are an efficient tool of multiuser communication, if the signals of individual users can be selected by a detector from their superposition at the channel output without any severe interference. The higher the signal-to-interference ratio, the closer the multiplex is to the ideal one. The approach to single-carrier (per user) CDMA historically evolved from Frequency Division Multiplex (FDM), through Time Division Multiplex (TDM) to the multiplex based on the general orthogonality of user-specific signatures – codes. This most contemporary multiplex is referred to as Code Division Multiplex (CDM) or Code Division Multiple Access (CDMA). The problem of such systems is preservation of orthogonality of multiplexed signals at the
receiver input. In case of orthogonal signatures of users, a bank of matched filters is the optimum linear receiver. But due to the character of a communication channel this orthogonality is usually impaired. Consequently, mutual interference of users cannot be completely canceled by the optimum linear receiver, either. This interference is referred to as multi-user interference (MUI) or multiple-access interference (MAI) [5]. In this case a decorrelating receiver and a bank of matched filters are asymptotically optimum solutions for high SNR and low SNR case, respectively. Minimum Mean Square Error (MMSE) receiver is an optimum solution in a transient cases in which neither influence of additive noise nor influence of interferences is strictly dominant [5], [9]

**Multicarrier code division multiplex**

Utilization of OFDM principle is not connected to the spread spectrum principle, as OFDM can be applied without any spectrum broadening. In such case the MC signal is a means for lowering the sensitivity of transmission to ISI caused by multipath propagation. Let us explain the approach using a few simple equations; this approach has been proved more “space effective”. Inclusion of figures into this contribution would increase its length substantially.

The starting point of MC system design are the parameters of the communication channel, its power delay profile and the delay-spread interval $T_{D\perp}$. Further main parameters of the channel are its bandwidth $B$ and its bandwidth of coherence $B_{COH}$. To suppress the influence of a delay spread, the symbol interval $T$ of MC modulation is chosen

$$T \approx 10 T_{D\perp}$$

Application of PSK/QAM modulation generates modulated carriers having $B_s = T^{-1}$ bandwidths. The second set of parameters is created by the signal and system parameters. Among them the ratio $N_{SP}$

$$N_{SP} = \frac{T}{T_b}$$

where $T_b = R_b^{-1}$ are the required bit interval and the transmission bit rate, respectively. MC transmission without spectrum spreading occupies the channel bandwidth

$$B_0 \approx \frac{1}{T_b} = N_{SP} \cdot \frac{1}{T} = N_{SP} \cdot B_s$$

Multiplexing of signals of multiple users calls for a much broader bandwidth, because only the increase of the signal space dimensionality makes the location of a higher number of orthogonal or weekly correlated signals possible. Let the total bandwidth of a channel be $B_{spread}$

$$B_{spread} = M \cdot B_0$$

Now, there are $N_p$ parallel channels

$$N_p = M \cdot N_{SP}$$

which can be utilized for multiuser exploitation. There are three basic multiplexing methods [6], [7]:

- Each user-specific signature has the number of chips equal to the number of carriers $N_p$. The transmitted data chips are not S/P converted, each bit is transmitted in all channels simultaneously, i.e., the symbol intervals of the channels have their initial values. Each symbol in the MC channel is multiplied by one chip of the signature. This multiplication factor is constant during the transmission by a specific user. The
signatures of individual users are mutually orthogonal. This method is called **frequency domain spreading or MC CDMA**.

- Each S/P converted symbol having its interval \( T = N_{sp} \cdot T_b \) is spread by its specific spreading modulation over the bandwidth \( M \cdot B_s \). Because there are \( N_{sp} \) such parallel channels, the total bandwidth of such signal is \( B_{spread} \) (16), (17). This method is referred to as **MC DS CDMA**.
- After S/P data conversion the MC OFDM signal is created. Its symbol interval is \( T \) and its bandwidth is \( B_0 \). The single user spreads the whole OFDM superposition specific spreading DS modulation. As individual modulated carriers have the frequency distance of adjacent sub channels \( \Delta = T^{-1} = B_s \), these signals have after spreading strongly overlapped spectra as each spread carrier has the bandwidth approx. \( M \cdot B_s \), i.e., M-times broader than the frequency distance. This signal pattern is usually referred to as the **multitone CDMA (MT-CDMA)**.

**Synchronization**

The advantageous features of spread spectrum signals applications in communication and measurements are reached at the expense of higher complexity of system realization. One of the most demanding processing is code synchronization, which is a method of determination of the a priori unknown time position of the spreading modulation (signatures) at the receiver input. Without this synchronization any despreading and demodulation of a received signal, i.e., any data transmission would not be possible. The code synchronization consists of two steps

- acquisition – a coarse time position estimation having error less than one chip interval
- tracking – minimizing of estimation error to a small fraction of a chip interval and tracking of time variations of the position

As the code synchronization has to be achieved in a very short time interval, sophisticated methods of code acquisition have been elaborated. This process has a specific mathematical model that differs from the classical formulation of parameter estimation. Instead of minimization of the error of an estimate based on a fixed extent of observation, the code position estimation does not minimize the error, but it minimizes the time of the estimation process which results in a limited error and the extent of observation is a free parameter or a sequence of parameters which has to be optimized. The error of the final estimate has to be lower than its prescribed maximum value.

For code tracking a delay tracking loop or tau dither tracking loop is utilized.

**Conclusions**

This contribution attempts to very briefly summarize an evolution of the area of spread spectrum communication systems including CDMA. Some topics, with which perhaps the targeted group of readers is not quite familiar, are dealt with in more detail. In contrast to the general practice the equations are used in this contribution for more precise description of some processes. This approach seems to be more concise than explanation using figures. These will be used in the oral presentation.
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References: